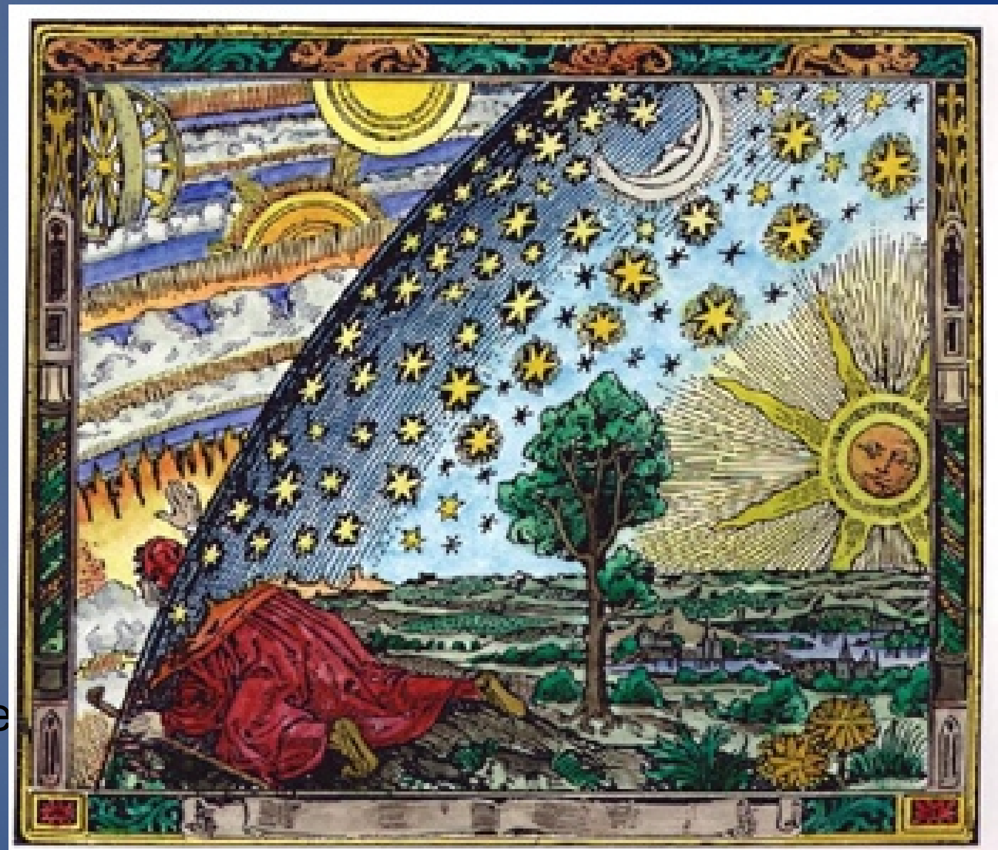
On Irreversibility and Cosmological Implications



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Outline

- Problem: an irreversible world for reversible dynamics
- Coarse graining, order, disorder
- Boltzmann entropy
- Molecular chaos and the Boltzmann equation
- Loschmidt and Zermelo paradoxes
- Time arrow: psychological aspects
- Cosmology and initial conditions

The vast majority of observed phenomena is characterized by a time asymmetry in its evolution. The seed give rise to the plant, not vice versa; heat flows from hot to cold bodies, not vice versa.

Our descriptions of macroscopic bodies successfully grasp this idea, in terms of macroscopic theories: thermodynamics, hydrodynamics etc.

The entropy of isolated systems must not decrease;
order cannot increase;

Energy given to system in order to do work (to produce a higher order) is eventually dissipated in the form of heat.

Since in the differential equations of mechanics themselves there is absolutely nothing analogous to the Second Law of thermodynamics the latter can be mechanically represented only by means of assumptions regarding initial conditions.

Ludwig Boltzmann

Atomistic hypothesis:

explain macroscopic behaviour from
microscopic laws of motion.

1. Hard to believe that time-symmetry breaking related to neutral kaons decay may determine behavior of macroscopic systems in standard temperature and pressure states.

2. Safe to assume irreversibility does not depend on internal structure of molecules, hence to restrict to translation degrees of freedom, adequately described by classical mechanics.

But classical (or quantum) mechanical laws assumed to govern evolution of particles, are time-reversal invariant (TRI).

Assume inter-particle forces *F* depend only on particles positions, Newton's equations for *N* particles system read:

$$m_i \frac{d^2 x_i}{dt^2} = F(x_1, \ldots, x_N), \quad i = 1, \ldots, N$$

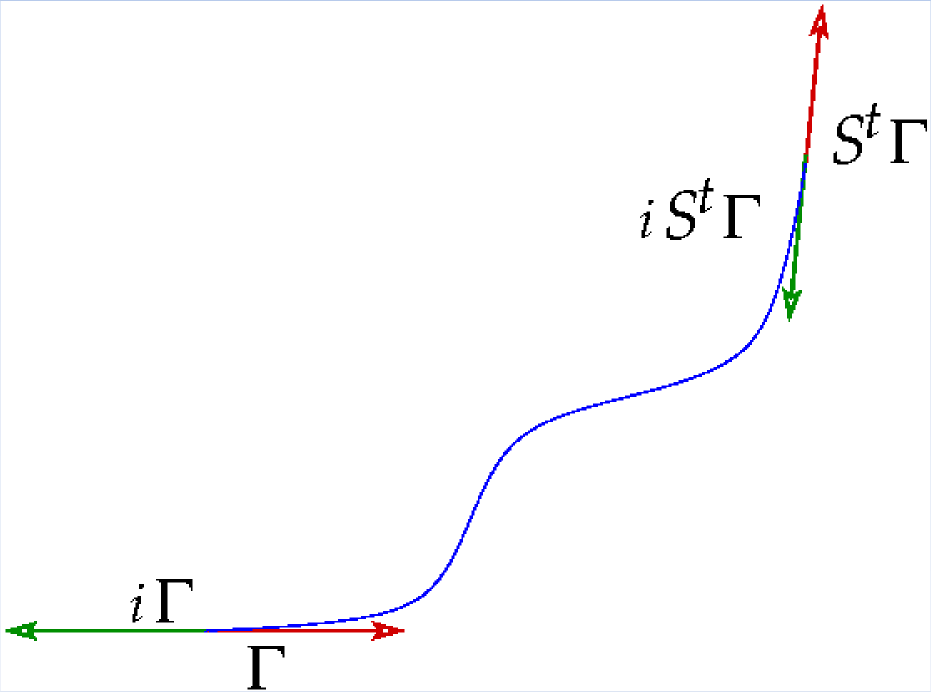
$\boldsymbol{\Gamma} = (x, v)$ **initial microscopic condition,**

S^t **evolution operator,**

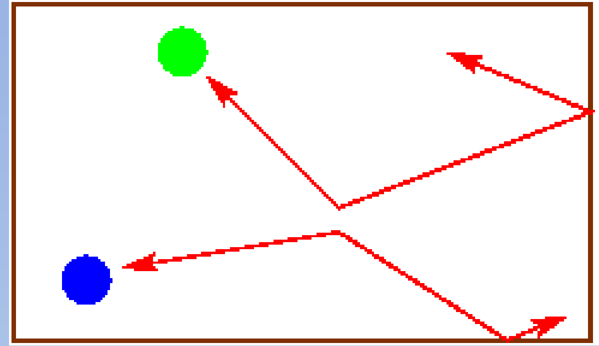
TRI implies that there exists time reversal operator i **such that:**

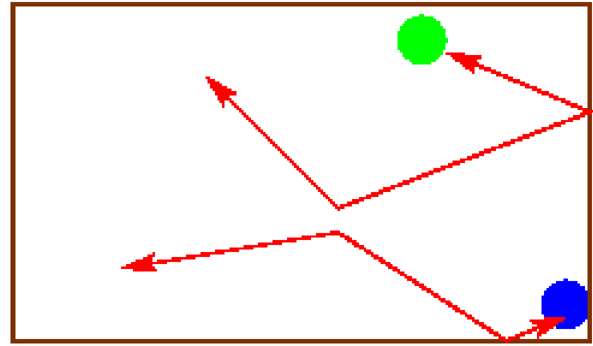
$$iS^t\,\Gamma = S^{-t} i\Gamma$$

Configurations traced back with opposite velocities (e.g. spin-echo)



Question: which picture has been taken first?





Reversibility of dynamics makes it impossible to answer, whether we look at the pictures or at the movie, because we don’t know whether the movie is being played forward or is rewinding.

The two processes are equally plausible.

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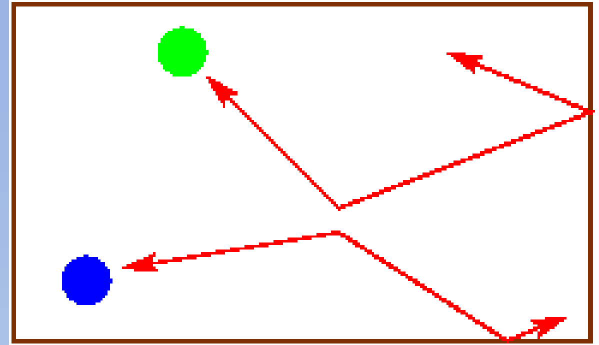

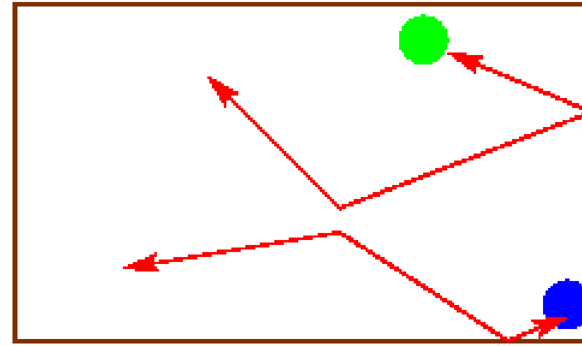

Reversibility of dynamics makes it impossible to answer, whether we look at the pictures or at the movie, because we don’t know whether the movie is being played forward or is rewinding.

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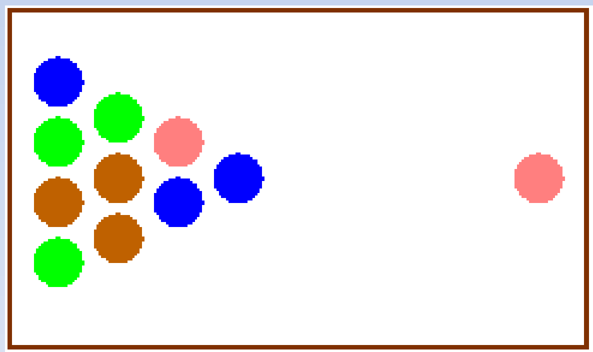
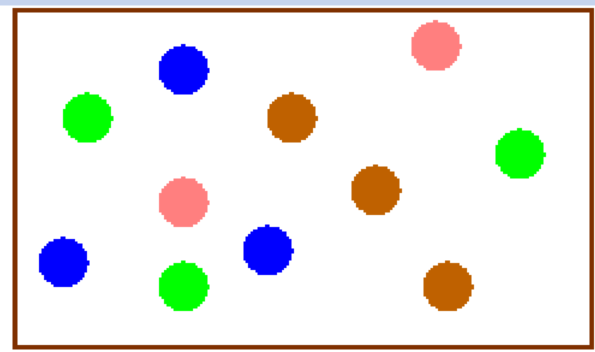
But let us consider a larger number of balls.

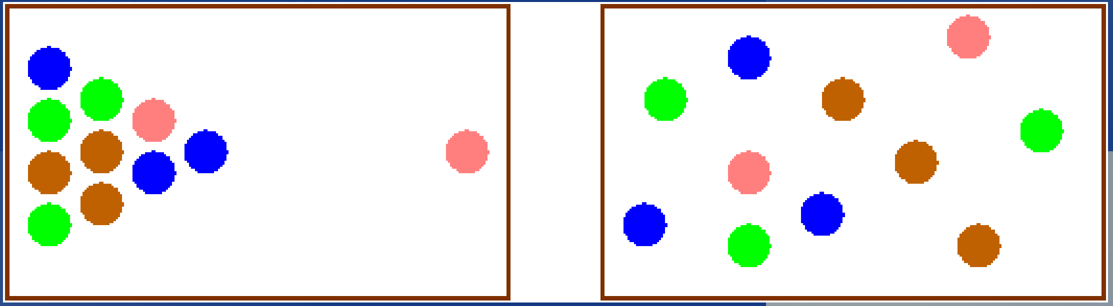




If we know that this is a spontaneous process, we have no doubts:

It suffices to look at the pictures; the movie is not necessary.

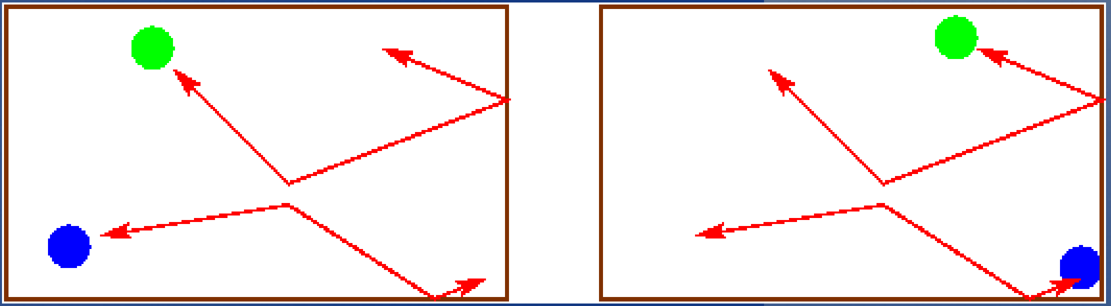
To obtain reverse motion, we would need 11 persons perfectly aiming separately at 11 balls, in contrast to a single person hitting randomly one ball. This is not impossible, it is not forbidden by any mechanical law, but the initial condition is so unlikely that we consider it unrealistic.



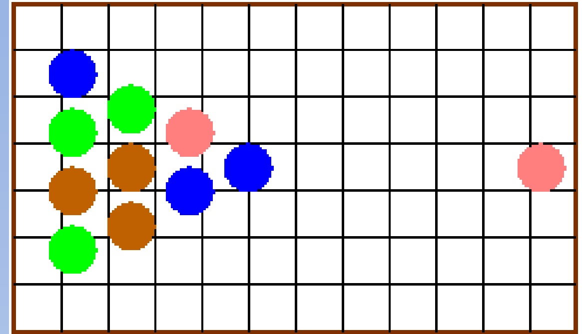
We have a sort of ***"practical irreversibility"*** given by the extreme difficulty of preparing an initial state leading from disorder to order.

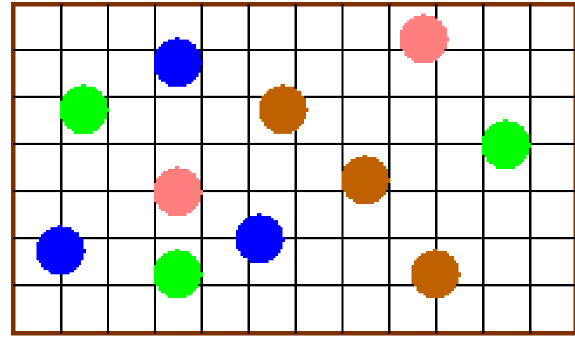
Interestingly, we can speak of natural motions going towards ***disordered*** states, as experienced in everyday life, only in dealing with large number of objects. With a few objects, notion of order or disorder make no sense.

Further, we think of disorder as ***randomness*** or ***uniformity***.



Subdivide table in 100
speeds in 100
speed directions in 50.
Take 11 balls.





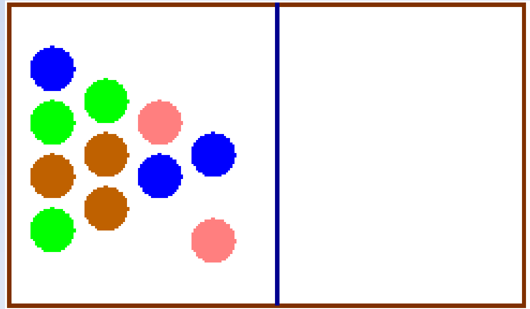
For simplcity, assume they interact very little (almost like points).
Then, phase space contains

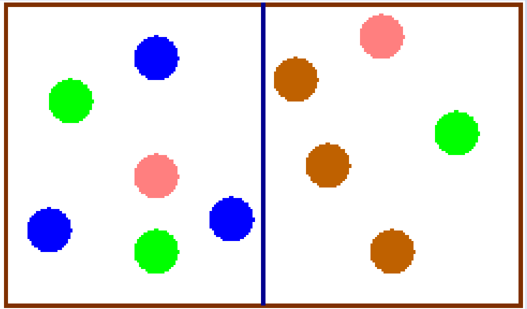
$$N = \frac{100!}{(100-11)!} 5000^{11} = O(10^{61})$$

possible configurations. One specific configuration, neglecting the order in which balls are taken, occupies *"only"* 11! of these configurations: aiming at our chosen configuration requires to hit 1 in $O(10^{7})/O(10^{61}) = O(10^{54})$ parts. Think of a case with 10^{24} particles!

Even with only 2 boxes: must

$$2^{10^{24}} \approx O(10^{72})$$

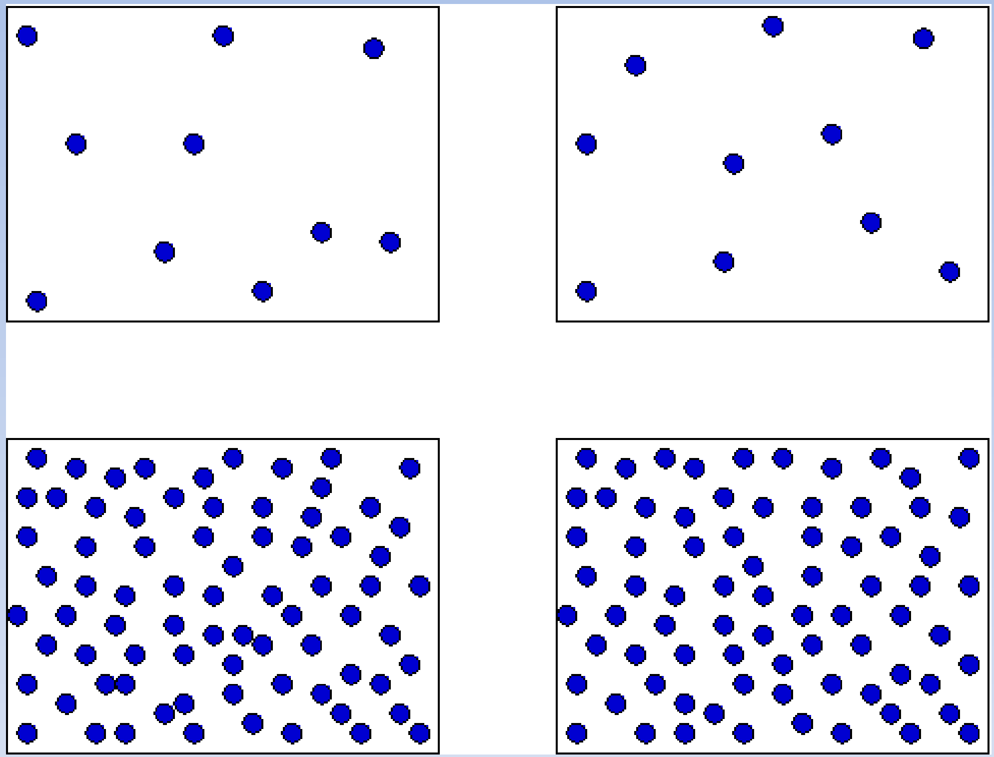




Utterly impossible!

This, however, is the same, whether we aim at a specific ordered or at a specific disordered configuration.

The fact is: we do not distinguish disordered configurations and the higher the number of particles the less we distinguish them.



If very many particles, most disordered configurations are indistiguishable: they are the same state!

This is quantified by the ***Boltzmann entropy***.

For an isolated thermodynamic system in a *microstate X* belonging to the class *M(X)* of all microstates with same value of a given *macroscopic* observable, let the **entropy** be defined by:

$$S_B(M) = k_B \log |M|$$

$|M|$ = fraction of microstates *X* in class of macrostate *M(X)*

$k_B = 1.38064 \cdot 10^{-23}\ JK^{-1}$

= remarkable Boltzmann constant.

$|M|$ typically grows in time, so does S_B. Maximum at equilibrium.

Note: *thermodynamic entropy* growth is not average property of an ensemble of macroscopic bodies, but of **EACH** macroscopic object.

Analogously, growth of the statistical mechanics *Boltzmann entropy* is not just an average growth:

it separately concerns **ALMOST ALL** microscopic states, or **i.e.**

Consider strings of *N* symbols: $S^N = \{(s_1 s_2 \ldots s_N)\}$ with $s_i \in A = \{a_1, a_2 \ldots, a_L\}$

$P(S^N) = \{\alpha = (\nu_1, \ldots, \nu_L)\}$ set of frequencies of symbols in strings of S^N

α a class. For instance: $A = \{a, b\}$

$$S^3 = \{aaa, aab, aba, baa, bba, bab, abb, bbb\}$$

$$P(aaa) = (1,0) \qquad P(aab) = P(aba) = P(baa) = \left(\frac{2}{3}, \frac{1}{3}\right)$$

$$P(bba) = P(bab) = P(abb) = \left(\frac{1}{3}, \frac{2}{3}\right) \qquad P(bbb) = (0,1)$$

8 microstates, 4 classes containing 1, 3, 3 and 1 elements.

View it as an idealization of *N* particles, each of which can take *L* configurations.

Different classes $< (N+1)^L$. Different strings L^N.

As N grows, a few classes become very large. If not small, to first order in the exponent, number of strings in one class is given by (combinatoric calculus and Stirling):

$$|\alpha| \approx \exp(NH(\alpha)) \quad \text{with} \quad H(\alpha) = -\sum_{k=1}^{L} \nu_k(\alpha) \ln \nu_k(\alpha)$$

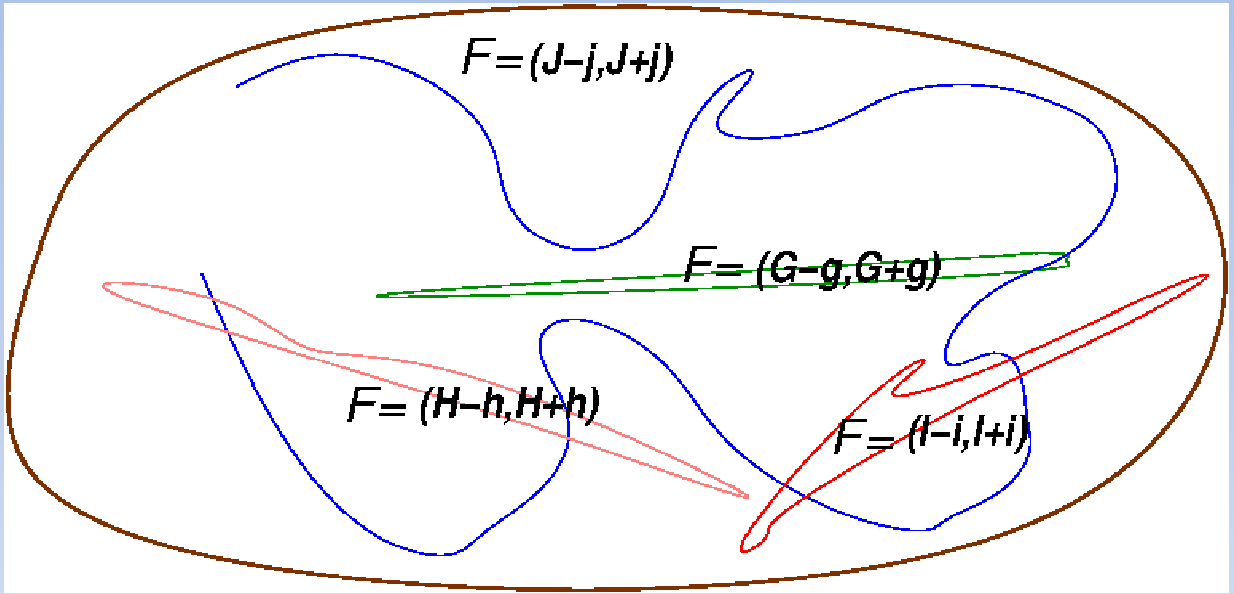
Large N: biggest class contains the ***vast majority*** of strings because microstates grow very rapidly with N, not so much the macrostates.

For system with macro-variable F (e.g. density), collect in one class states with same value of F within tolerance. If one class much larger than others, and *all states have equal probability,* F quickly becomes practically constant around that class value, microstate evolves remaining almost all its time in that equilibrium state.

Irreversibility emerges as a property of the evolution of global variables in systems made of very many particles.

Khinchin says: this allows us

“to represent the mean values of sum functions, and permits us to identify them with time averages which represent the direct results of any physical measurement.”



Phase space partition:

macrostate specified by global variables and tolerances yields coarse grained description of microstates, sufficient to macroscopic purposes:

evolution of observables = entering different (non-local) regions

To quantify the evolutions of a gas towards uniform density, against evolutions away from that, let N_R particles on right, N_L on left. Equilibrium is given by:

$$N_L = \frac{N}{2} = N_R$$

Begin with $N_R(0) < N/2$ and count states:

$\Gamma_{++}\left(N_R\right)$ = states at time 0 for which $N_R(-dt) < N_R(0) < N_R(dt)$

$\Gamma_{--}\left(N_R\right)$ = states at time 0 for which $N_R(-dt) > N_R(0) > N_R(dt)$

$\Gamma_{-+}\left(N_R\right)$ = states at time 0 for which $N_R(-dt) > N_R(0) < N_R(dt)$

$\Gamma_{+-}\left(N_R\right)$ = states at time 0 for which $N_R(-dt) < N_R(0) > N_R(dt)$

In the large N limit, one has a function $g(N) \underset{N\to\infty}{\to} 0$ such that

$\Gamma_{++}(N_R)$ = evolving towards equilibrium forward in time = $g(1-g)$

$\Gamma_{--}(N_R)$ = evolving away from equilibrium forward in time = $g(1-g)$

$\Gamma_{-+}(N_R)$ = closer to equilibrium in past as well as in future = $(1-g)^2$

$\Gamma_{+-}(N_R)$ = farther from equilibrium in past as well as in future = g^2

Almost all initial conditions get closer to equilibrium in future

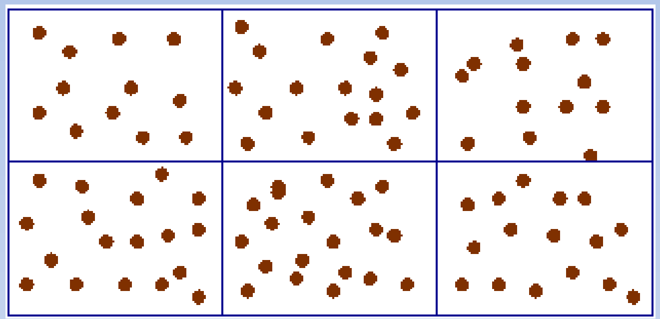
$$\Gamma_{++}(N_R) + \Gamma_{-+}(N_R) = 1 - g$$

Only $\Gamma_{--}(N_R) + \Gamma_{+-}(N_R) = g$ get farther from equilibrium in future

But almost all initial conditions get closer to equilibrium both in past and in future: $\Gamma_{-+}(N_R) = (1-g)^2$

Boltzmann considered dilute gas of particles interacting with short range potentials. Potential energy is negligible, compared to kinetic energy, but fundamental for thermodynamics to be established.

One relevant observable is the mass density distribution



$$f(x_i, p_j, t)\Delta x \Delta p = n_{ij}$$

$$\int f(x, p, t)\,dx\,dp = N$$

$$\left(\frac{\partial}{\partial t} + \frac{p}{m}\cdot\nabla_x\right) f(x, p, t) = \left(\frac{df}{dt}\right)_{coll}$$

Without collision term, the Boltzmann equation looks like the Liouville equation in phase space, but:

- This is in *6* dimensions, not *6N* dimensions
- Particles, not points, occupy space; statistics needs $N >> n_j >> 1$

Boltzmann's crucial assumption: ***"molecular chaos"***:

momenta of particles interacting around *x*, are independent:

$$\left(\frac{df}{dt}\right)_{coll} =$$

$$\int dp_1' dp' dp_1 F(x, p, p', p_1, p_1') \cdot \left[f(x, p', t) f(x, p_1', t) - f(x, p, t) f(x, p_1, t) \right]$$

p', p_1' = momenta of two particles before collision

p, p_1 = momenta of two particles after collision

In contrast to Newton's equation for microstates, Boltzmann eq. for a macrostate, is not invariant under time reversal.

$t \rightarrow -t$ and $p \rightarrow -p$ yields: $\left(\frac{df}{dt}\right)_{coll} \mapsto -\left(\frac{df}{dt}\right)_{coll}$

Introduce Boltzmann's H-functional:

$$\mathcal{H}(t) = k_B \int dxdp\, f(x,p,t) \log f(x,p,t)$$

which equals $-S_B(t) = -k_B \log\left(\left\| f(t) \right\|\right)$ in the case of a rarefied gas not too far from equilibrium.

Because of the irreversibility of the Boltzmann equation, its solutions obey the
H-theorem

$$-\frac{d\mathcal{H}}{dt}(t) = \frac{dS_B}{dt}(t) \geq 0$$

"=" if and only if f = Maxwell–Boltzmann equilibrium distribution.

Theorem reflects irreversible (time-asymmetric) character of Boltzmann equation; it derives from molecular chaos assumption: distribution gets smoother forward in time!

The “reversibility objection” or “Loschmidt paradox”:

H-theorem cannot be a consequence of reversible microscopic mechanics: if $\mathcal{H}$ decreases in time, a reversal of velocities of all atoms yields an initial condition for an increase of $\mathcal{H}$.

The “recurrence objection” or “Zermelo paradox”

Based on Poincaré recurrence theorem: given any tolerance, a mechanical system with bounded phase space takes a finite time T_R to return to its initial condition within that tolerance. If $\mathcal{H}$ initially decreases, it must increase again within the time T_R.

Boltzmann himself noted that T_R for a macroscopic system is, however, extremely long: for instance $10^{10^{19}}$ years for 1 cm^3 of gas in normal conditions. Universe only 10^{10} years: should not enjoy same physical properties over such a long time.

In practice, the answer to all these questions is this figure for the *H-functional*:

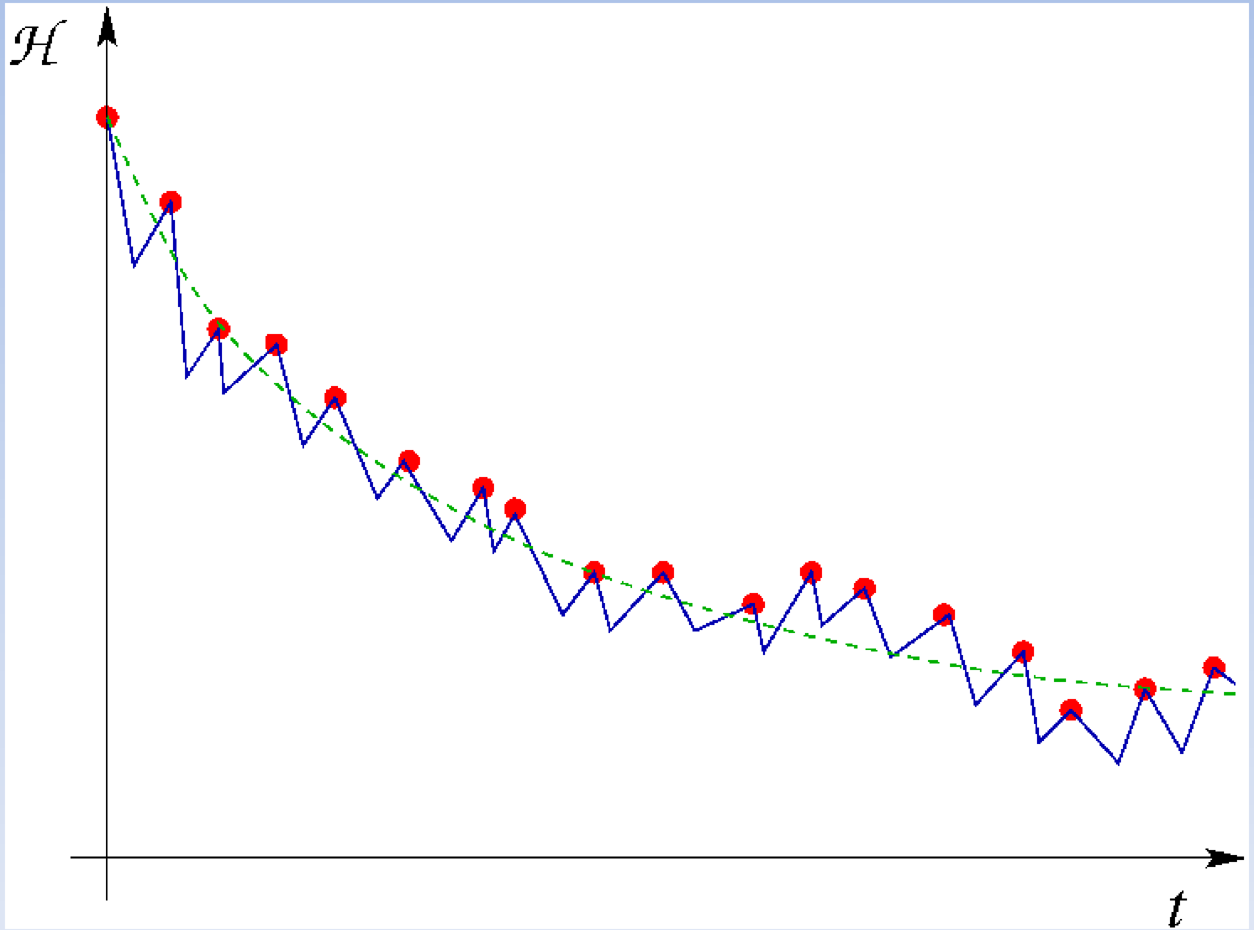
peaks are states in which molecular chaos holds.

As N grows, peaks become denser.

In the limit, *H-functional* is monotonic and follows smooth curve.

This explains the irreversibility of a dilute gas in an isolated box.

In general? Our Universe is much more complex!



How come future looks to us so very differenty from past?
We see present caused by past & causing future, never the opposite.
Why are we so sure that time really does go? ***The TIME ARROW!***
If a dice rests on a table, we do not know how it got there; knowledge of its state is too coarse: we neglect the details of its microscopic phases, necessary to know its past. On the contrary, we can predict that it will stay there, if the forces acting on it sum to 0.

We can't travel backward in time, so no interest in acting on past, while we want to have a better future.

Reichenbach, thought that such asymmetry is due to the fact that we search for ***relations*** among things. A footprint in the sand is never interpreted a physically possible ***spontaneous large deviation***; immediately we assume that someone walked on the sand, as we deem unreasonable the *probability* of such a large deviation.

Is the distinction between past and future just ***psychological?***

Einstein's friend M. Besso passed away a few months before Einstein, who then wrote a moving letter to his friend's widow and son:
Michele has preceded me a little in leaving this strange world. This is not important. For us who are convinced physicists, the distinction between past, present and future is only an illusion, however persistent (cited by Prigogine)

Popper rejected this view. Was Hiroshima an illusion?
Question remains. How far are we from an answer?

In 1905, Boltzmann thought that one day his atomistic hypothesis could perhaps be disproved and matter be better described by a continuum. He almost regretted that one should die before the question could be settled: *"How immoderate we mortals are! Delight in watching the fluctuations of the contest is our true lot."*

The very same years 1905 - 1908 dispelled all doubts!

Cosmology and Thermodynamics

Eddington popularized the link between entropy growth and the ***arrow of time***: **The arrow telling us the direction of time.**

Above would explain arrow if Universe was like a gas in isolated box.
Much more complex. Must admit many boxes, at least, in some entropy increases, in others decreases in time. Why do all observations agree with same arrow?

Systems not really isolated, entropy increase applies only to entire universe (if mechanical): arrow the same for all its parts.

Very distant bodies exchange energy only by radiation, which tends to leave rather than arrive (Olbers's paradox). Universe quite different From one box full of gas: it does not seem to reflect radiation.
It is expanding and matter moves away from matter extremely rapidly
Expansion indicates privileged direction of time: **real time arrow!**
(Do not confuse with entropy increase for expanding gas)

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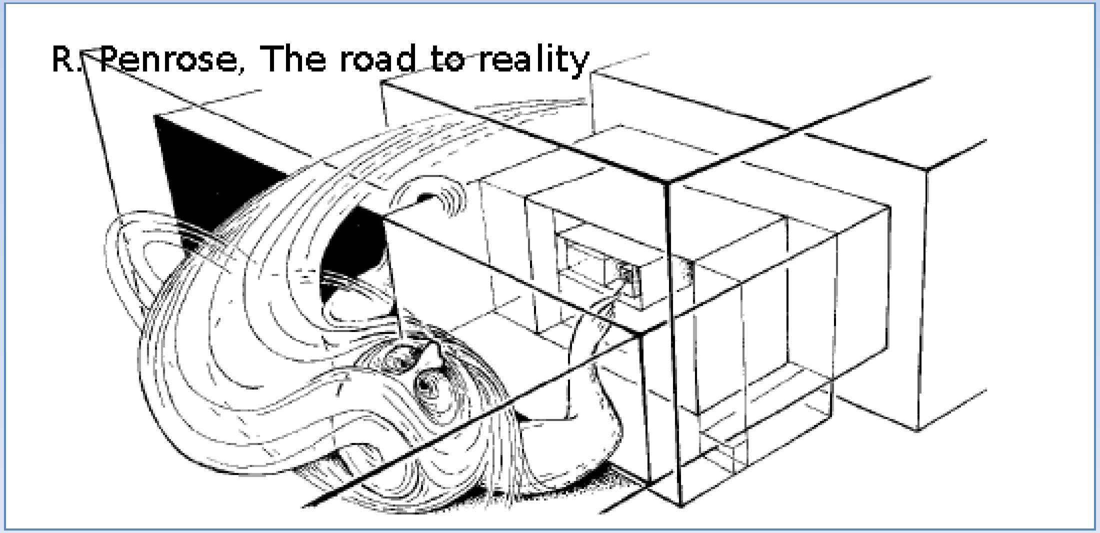
Then, if the entropy of our Universe always grows, it must have started from a very low value, i.e. in a very small region. Penrose estimates it to be the ridiculous number: ***one part in*** $10^{10^{123}}$ **!**

That would allow entropy growth for ultra-astronomic times.

Very special initial condition. As Newton noted, sole law of nature does not suffice, mechanics needs initial conditions:

"blind fate cannot make planets move in a single and same fashion in concentric orbits. This uniformity must be due to a choice."

Indeed, the low probability of an initial condition for which the entropy decreases is not forbidden by any known law of physics.



C. Callender: **i.c. should be introduced as a law itself!**