

**Distance, dissimilarity index, and network community structure**

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We address the question of finding the community structure of a complex network. In an earlier effort [H. Zhou, *Phys. Rev. E* **67**, 041908 (2003)], the concept of network random walking is introduced and a distance measure defined. Here we calculate, based on this distance measure, the dissimilarity index between nearest-neighboring vertices of a network and design an algorithm to partition these vertices into communities that are hierarchically organized. Each community is characterized by an upper and a lower dissimilarity threshold. The algorithm is applied to several artificial and real-world networks, and excellent results are obtained. In the case of artificially generated random modular networks, this method outperforms the algorithm based on the concept of edge betweenness centrality. For yeast's protein-protein interaction network, we are able to identify many clusters that have well defined biological functions.

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**I. INTRODUCTION**

A graph (network) of vertices (nodes) and edges is a useful tool in describing the interactions between different agents of a complex system. For example, if we want to analyze protein-protein physical interactions in yeast *Saccharomyces cerevisiae* [1], we would like to denote each protein as a distinct vertex of a graph, and setup an edge between two vertices if the corresponding proteins have direct physical interactions. Many such kinds of networks are constructed in sociological, biological, and technological fields, and they usually have very complicated connection patterns. What one needs is a method that is capable of classifying vertices of a complex network into different clusters (communities). If a network is appropriately decomposed into a series of functional units, (a) the structure of the network can be better understood and the relationship between its different components will be clear, (b) the principal function of each cluster can be inferred from the functions of its members, and (c) possible functions for members of a cluster can be suggested by comparing the functions of other members. Network clustering techniques are therefore very important in the emerging fields of bioinformatics and proteomics.

A good clustering method needs to satisfy two conditions: First, the inherent structure of the network should be reserved; second, it should provide a quantified resolution parameter to mark the significance of the clusters obtained at each level of the partitioning process. The global organization of a network should already be identified at low resolutions and more and more fine structures emerge as the resolving power is increased.

Many existing methods [2,3] only take account of local information of each vertex, such as number of nearest neighbors shared with other vertices, number of vertex-independent paths to other vertices, etc. Recently, Girvan and Newman [4] suggested an elegant global algorithm that extended the concept of vertex betweenness centrality of Freeman [5] also to edges. Their algorithm works iteratively by removing the current edge(s) of the highest degree of betweenness centrality. When applying to an ensemble of random modular networks, this algorithm greatly outperforms

some conventional methods [4]. On the other hand, it does not provide a parameter to quantify the differences between communities; furthermore, the concept of edge betweenness centrality is most natural for unweighted networks.

In Ref. [6] a Brownian particle is "introduced" into a network to "measure" the distances between vertices. In the present work, we extend the basic idea of Ref. [6] by defining, based on this distance matrix, a quantity called the dissimilarity index between nearest-neighboring vertices. The dissimilarity index signifies to what extent two nearest-neighboring vertices would like to be in the same community. A hierarchical algorithm is then worked out; it makes use of information on the dissimilarity indices and decompose a network into a hierarchical sequence of clusters. Each of the communities is characterized by an upper and a lower dissimilarity threshold.

The method, which could work on unweighted as well as weighted networks, is applied to several artificial and real networks, and very satisfying results are obtained. For the case of random modular networks, the present algorithm outperforms the method of Girvan and Newman [4]. When applying the algorithm to the protein-protein interaction network of yeast, we are able to identify many protein clusters that have well defined biological functions.

In Sec. II, we review the distance measure of Ref. [6] and define a dissimilarity index for each pair of nearest-neighboring vertices. A dissimilarity-index-based hierarchical algorithm is outlined in Sec. III, and applied to two kinds of artificially generated networks and four real-world networks in Sec. IV. We conclude our work in Sec. V with a brief discussion.

**II. DISTANCE MEASURE AND DISSIMILARITY INDEX**

In the opinion of Flake, Lawrence, and Giles [7], a community in a (sub)graph should satisfy the requirement that each vertex's total intracommunity interaction be stronger than the total interaction with other vertices in the (sub)graph. This turns out to be a very strong constraint. In this work, we weaken this condition and require only that a vertex should have stronger total interaction with other ver-

tices of its own community than with vertices of any another community of the (sub)graph.

We consider a connected network of  $N$  vertices and  $M$  edges. The network's connection pattern is specified by the generalized adjacency matrix  $A$ . We assume that the value of each nonzero element of matrix  $A$  (say  $A_{ij}$ ) denotes the interaction strength between vertex  $i$  and  $j$ . The distance  $d_{ij}$  from vertex  $i$  to vertex  $j$  is defined as the average number of steps needed for a Brownian particle on this network to move from vertex  $i$  to vertex  $j$  [6]. At each vertex (say  $k$ ) the Brownian particle will jump in the next step to a nearest-neighbor vertex (say  $l$ ) with probability  $P_{kl} = A_{kl} / \sum_{m=1}^N A_{km}$ . The distance matrix thus defined is asymmetric (in general,  $d_{ij} \neq d_{ji}$ ), and it is calculated by solving  $N$  linear-algebra equations [6].

Taking any vertex  $i$  as the origin of the network, then the set  $\{d_{i1}, \dots, d_{i,i-1}, d_{i,i+1}, \dots, d_{iN}\}$  measures how far all the other vertices are located from the origin. Therefore, it is actually a perspective of the whole network with vertex  $i$  being the viewpoint. Suppose vertices  $i$  and  $j$  are nearest neighbors ( $A_{ij} > 0$ ), the difference in their perspectives about the network can be quantitatively measured. We define the dissimilarity index  $\Lambda(i, j)$  by the following expression:

$$\Lambda(i, j) = \frac{\sqrt{\sum_{k \neq i, j}^N [d_{ik} - d_{jk}]^2}}{(N-2)}. \quad (1)$$

If two nearest-neighbor vertices  $i$  and  $j$  belong to the same community, then the average distance  $d_{ik}$  from  $i$  to any another vertex  $k$  ( $k \neq i, j$ ) will be quite similar to the average distance  $d_{jk}$  from  $j$  to  $k$ , therefore the network's two perspectives (based on  $i$  and  $j$ , respectively) will be quite similar. Consequently,  $\Lambda(i, j)$  will be small if  $i$  and  $j$  belong to the same community and large if they belong to different communities.

### III. THE ALGORITHM

We exploit the dissimilarity index to decipher the community structure of a network. After the distance matrix  $\{d_{ij}\}$  and the dissimilarity indices for all the nearest-neighbor vertices  $\{\Lambda(i, j)\}$  are obtained, the algorithm works as follows.

(1) Initially the whole network is just one single community. This community is assigned an upper dissimilarity threshold  $\theta_{\text{upp}}$  equaling to the maximum value of all the different dissimilarity indices.

(2) For each community, a resolution threshold parameter  $\theta$  is introduced and is assigned the initial value  $\theta_{\text{upp}}$  of that community. The algorithm is unable to discriminate between two nearest-neighbor vertices  $i$  and  $j$  when  $\Lambda(i, j) \leq \theta$ ; if this happens, vertices  $i$  and  $j$  are marked as "friends."

(3) The  $\theta$  value is decreased differentially. All edges in the community are examined to see whether two nearest-neighbor vertices are friends. Different friends sets of the community are then formed, each of which contains all the friends of the vertices in the set. There may also be vertices in the community that do not have any friends. Each of these

vertices is moved to the friends set that, according to the generalized adjacency matrix  $A$ , has the strongest interaction with it. After this operation, vertices of the community are distributed into a number of disjointed sets (this number may be unity).

(4) Each vertex in a subcluster should have stronger interaction with vertices within this subcluster than with vertices of any another subcluster of this community. To fulfill this requirement, we perform a local adjustment process, move each of the vertices that fail to meet this requirement to the friends set that has the strongest total interaction with it. This adjustment process is performed simultaneously for all these unstable vertices and is repeated until no unstable vertices remains.

(5) If vertices of the community remain together, the algorithm returns to step (3). If they are divided into two or more sets, then the community under processing is assigned a lower dissimilarity threshold  $\theta_{\text{low}}$  equaling to the current  $\theta$  value, and it is no longer considered. Each of the identified subsets of this community is regarded as a new (lower-level) community, with upper dissimilarity threshold  $\theta_{\text{upp}}$  equaling to the current  $\theta$  value. The algorithm returns to step (2) to work with another identified community.

(6) After all the (sub)communities are processed, a dendrogram is drawn to demonstrate the relationship between different communities as well as the upper and lower dissimilarity thresholds of each community. The vertex set of each community is also reported.

The above procedure could be easily implemented with C++ programming language. The source code as well as the data for the examples studied in the following section will be made publicly available [8].

## IV. APPLICATIONS

We test the performance of the above-mentioned algorithm by applying it first to two kinds of artificial networks and to three real-world networks.

### A. Artificial random modular networks

To quantitatively compare with the work of Girvan and Newman [4], the algorithm is first applied to a random modular network. The network has 128 nodes, which are divided into four modules of size 32 each. Each vertex has, on average, 16 edges connecting to other vertices, and on average  $\bar{z}_{\text{out}}$  of each vertex's edges are connected to vertices of other modules. All the edges are setup randomly with these two fixed expectation values. The present method is able to recover the modular structure of the network up to  $\bar{z}_{\text{out}} \approx 7$ . It slightly outperforms the method of Girvan and Newman [4] in performance. For example, working on an ensemble of random graphs with  $\bar{z}_{\text{out}} = 6.0$  by the present method, on an average only 4.5 vertices are misclassified, each of which is assigned a cluster identity different from those of the majority of vertices of its module; while on an average about 13 vertices are misclassified by the method of Girvan and Newman [4].

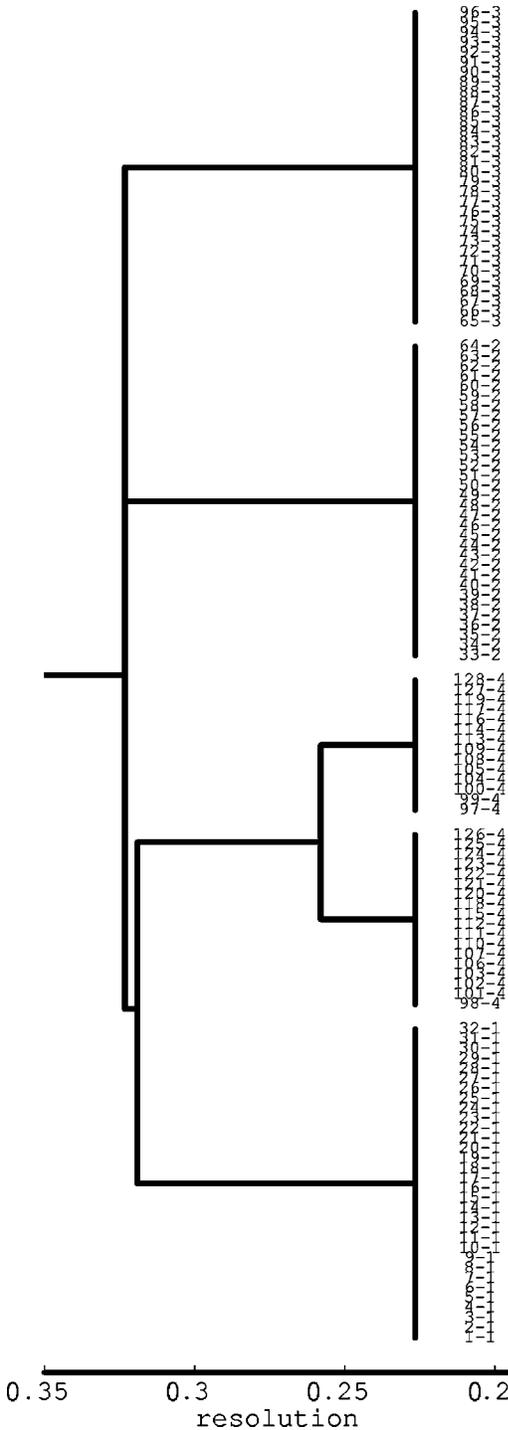


FIG. 1. The community structure of a random modular network of 128 vertices and 1067 unweighted edges (see the main text for the rules how such a network is generated). Here and in following figures, in the pattern  $xx-yy$ , the number  $yy$  after the hyphen denotes the group identity of vertex  $xx$  according to information from other sources.

In Fig. 1, the community structure of a randomly generated modular network with  $\bar{z}_{out} = 6.0$  is demonstrated. When the resolution threshold is beyond 0.323, the network as a whole could be regarded as a giant community. At resolution threshold 0.323, however, three subgroups suddenly emerge,

with size 32, 32, and 64, respectively. The first two communities correspond to two modules of the network, and the last one is the combination of the other two modules. At resolution threshold 0.319, this later community again is divided into two subcommunities of 32 vertices each, corresponding to the remaining two modules. At resolution threshold 0.258, one of the modules of the network is found to fission into two subgroups of size 14 and 18, respectively. In this example, the designed four modules of the network correspond to the resolution range from 0.258 to 0.319.

How to interpret the resolution parameters in the dendrograms such as that shown in Fig. 1? Take module 2 and module 3 as examples. Figure 1 suggests that edges between these two modules have dissimilarity indices larger than 0.323, while edges within these modules have dissimilarity indices  $\approx 0.227$ . Therefore there is a large dissimilarity gap of about 0.1 between an intermodular edge and an intramodular edge.

It is noticeable that by the present algorithm, each community has certain range of stability. Subcommunities emerge only when the resolution threshold is lowered below certain level, and they emerge abruptly.

### B. Regular hierarchy networks

We analyze here the community structure of the model hierarchy network studied by Ravasz and co-workers [3]. The network is constructed by several steps [3]. At level  $n = 0$ , a fully connected unit of four vertices is generated. At level  $n = 1$ , three replicas of this unit are added and the external vertices of these replicas are connected to the central vertex of the  $n = 0$  unit, while the central vertices of the replicas are connected to each other. This replication-connection process could be continued to any desired level  $n$ . In Fig. 2(a) such a network at level  $n = 2$  is shown. It was remarked [3] that conventional network clustering methods are unable to uncover the hierarchical structure of such a network. The present method, however, works very well, Fig. 2(b) demonstrates the obtained community structure of the network in Fig. 2(a). The hierarchy organization of the vertices in the network is largely reserved in Fig. 2(b). At resolution threshold 1.95, the network is divided into four subgroups of size 3 and a giant component of size 62. Later at resolution threshold 1.89, this giant component again is fissioned into two parts: one part has size 12 and is further divided into three subgroups of size 4 at resolution threshold 1.52; the other part has size 50, which at resolution threshold 1.53 further decomposes into three subgroups of size 13, 13, and 14, respectively. At resolution threshold 0.91, each of these three subgroups is further divided into three subgroups.

### C. The karate club network

The karate club data [9] examined in Refs. [4,6] is re-evaluated here. This network is weighted, each edge is assigned a different strength. The present algorithm leads to the community structure of Fig. 3. At resolution threshold 1.67, the network decomposes into one small component  $A$  of five vertices and a large component of 29 vertices. At resolution

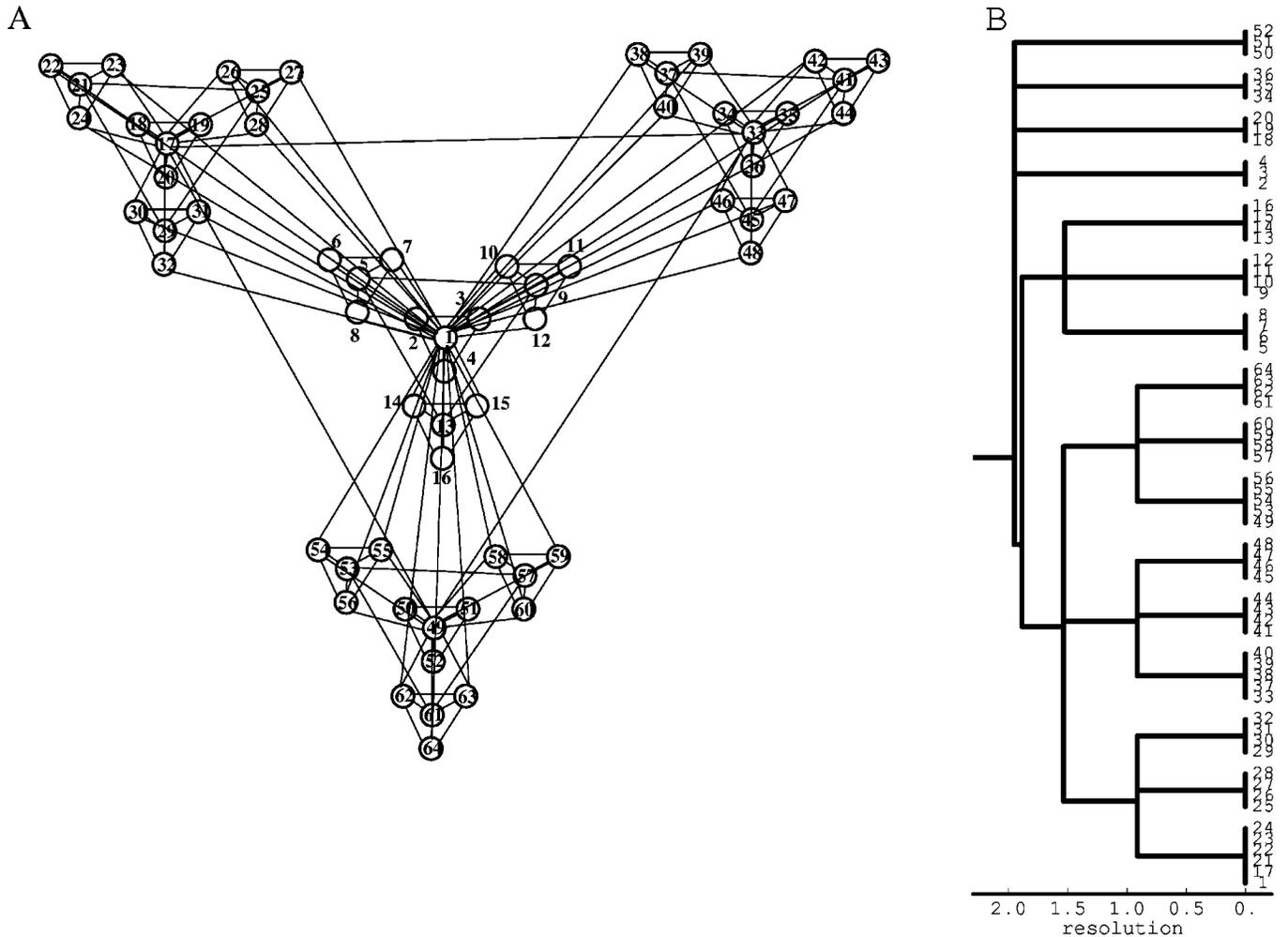


FIG. 2. A hierarchy network [3] at level  $n=2$  (a) and its community structure (b).

threshold 0.87, this large component further decomposes into two subgroups, one of which has 11 members (*B*) and the other has 18 members (*C*). Comparison with the actual fission pattern is also shown in Fig. 3. According to Fig. 1(a) of Ref. [6], the connection pattern of the three components is linear with  $A-B-C$  (there is no edges between *A* and *C*). The algorithm of Ref. [4] first cuts edges between *B* and *C*, and then cuts edges between *A* and *B*; in Fig. 3 this order is reversed. When several components are linearly connected, their hierarchical organization may be a little bit arbitrary.

**D. The football team network**

The football team network compiled by Girvan and Newman [4] and studied in Refs. [4,6] is reinvestigated here. The present method results in the community structure of Fig. 4. Each vertex's actual group identity is also shown for comparison. In the resolution region between 0.41 and 0.64 there are 12 communities according to the present algorithm. Of the 12 actual groups, only members from group 12 are distributed to other groups (with good reasons, because actually there are very few direct interactions between the five members of this cluster). Vertex 111 are classified together with members of group 11, we have checked that this vertex has eight edges linking to group 11 and only three edges to other

groups. Vertex 59 is classified together with members of group 9, we have also checked that it has stronger interaction with group 9 than with any another group.

The organization of the different teams suggested by the present algorithm seems to be even better than the original organization.

**E. The scientific collaboration network**

The scientific collaboration network compiled by Girvan and Newman [4] and examined in Refs. [4,6] is also reexamined. This network is also weighted [see Fig. 5(a)]. The present method suggests a community structure shown in Fig. 5(b). In accordance with the actual situation, on the global scale, the network clearly has three giant communities of comparable sizes. Each of these giant communities could further be decomposed into several subcommunities when the resolving power is increased.

**F. The protein interaction network of yeast**

The protein interaction network of yeast is constructed based on the data reported in Refs. [10,11], it contains 1471 proteins and 2770 edges (protein-protein physical interactions). This network has already been studied in Ref. [6];

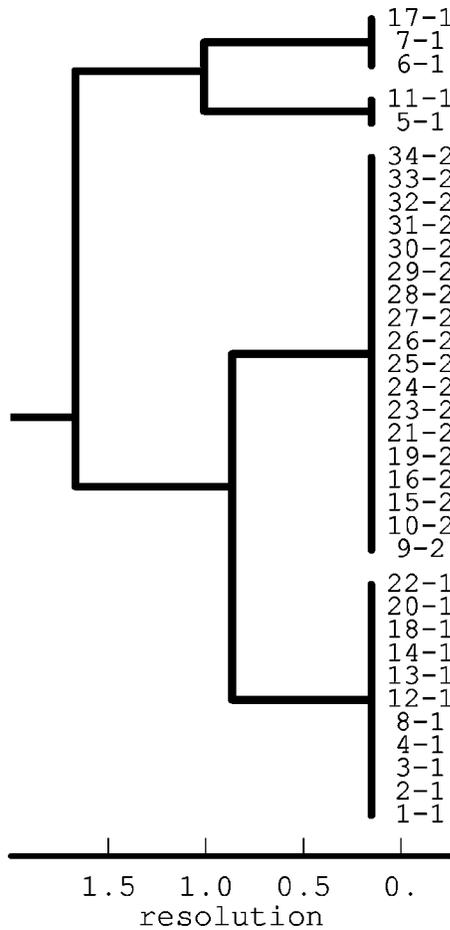


FIG. 3. The community structure of the karate club network of Zachary [9].

here we constructed a reduced interaction network based on the original one. First, self-connection is removed; second, proteins that are connected to the network by only one edge are removed. The second step is continued until no proteins of degree one remain. The reason to remove all the proteins of degree one is that, according to the idea of Girvan and Newman [4], a vertex that is connected to the network by just one edge should be in the same community as its nearest-neighboring vertex, therefore its status need not to be considered separately. Of course, we have checked that actually identical results are obtained when the network-reduction process is not performed. The reduced network contains 871 proteins and 2043 unweighted interactions (edges).

The community structure of this network is demonstrated in Fig. 6. It seems to be strikingly different from those of the other networks studied in this paper. At the resolution range between  $\sim 1.5$  and 18.0 many small communities appear, but the network is dominated by just one large cluster of size proportional to the total size of the network. This is in accordance with Ref. [6] where the original network was decomposed into one large component and several small components. When the resolution threshold is decreased below 1.5, the largest cluster is divided into several subclusters of comparable sizes. The biological significance of such a community structure is yet to be investigated.

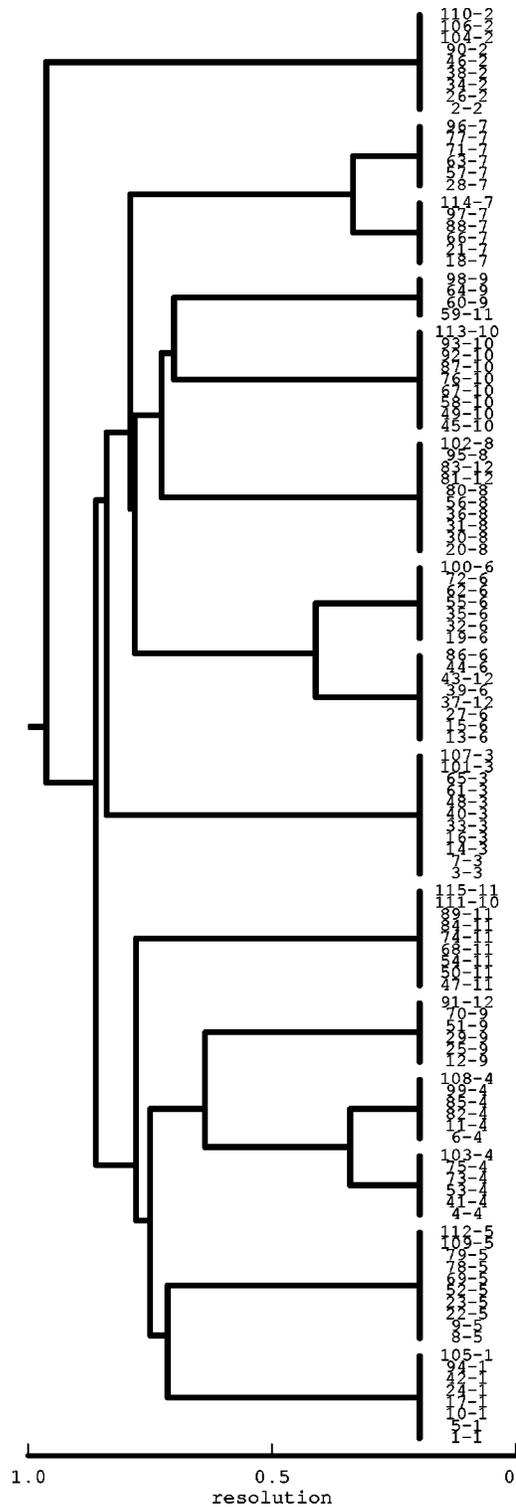


FIG. 4. The community structure of the football network of Girvan and Newman [4].

Based on the community structure shown in Fig. 6, we can construct clusters of proteins that might be of biological significance. For the purpose of demonstration, here we show three examples of such protein clusters, corresponding, respectively, to higher, medial, and lower resolution thresholds.

The first example is a cluster that appears at resolution

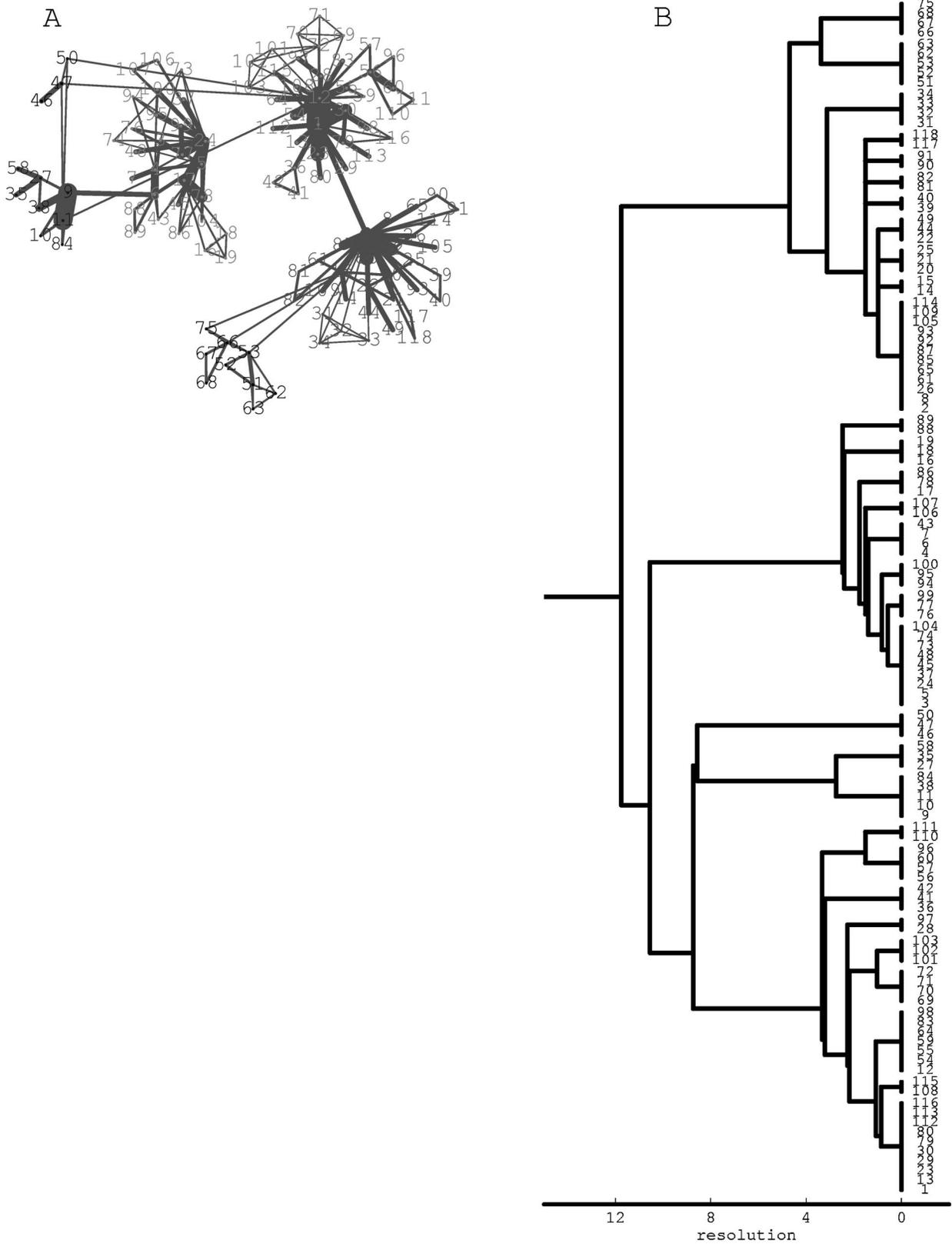


FIG. 5. (Color online) The scientific collaboration network (a) compiled by Girvan and Newman [4] and its community structure (b). In (a) the different communities emerging at resolution threshold 4.0 are demonstrated.



FIG. 6. The community structure of the reduced protein-protein interaction network of yeast.

threshold 18.04. It contains 16 proteins and 33 edges, and has the structure shown in Fig. 7(a). This cluster is stable, namely, that each vertex in it is more connected to vertices in this cluster than to vertices outside; and it has no further subcommunity structure. According to the protein interaction databank [10,12], 15 of these proteins are all involved in adenosine triphosphate (ATP) synthesis process in yeast. They may form a very important part of yeast's mitochondrial ATPase complex. One protein of this cluster, YIL124W, is a hypothetical membrane protein. Because this last protein has only one interaction with other members of the cluster, it may not have similar biological functions as the other members.

The second example is a cluster that appears at resolution threshold 5.11. It contains 11 proteins and 38 edges, and has the structure shown in Fig. 7(b). This cluster is also stable and has no further structure. According to the protein interaction databank [10,12], among these 11 proteins, YBL084C,

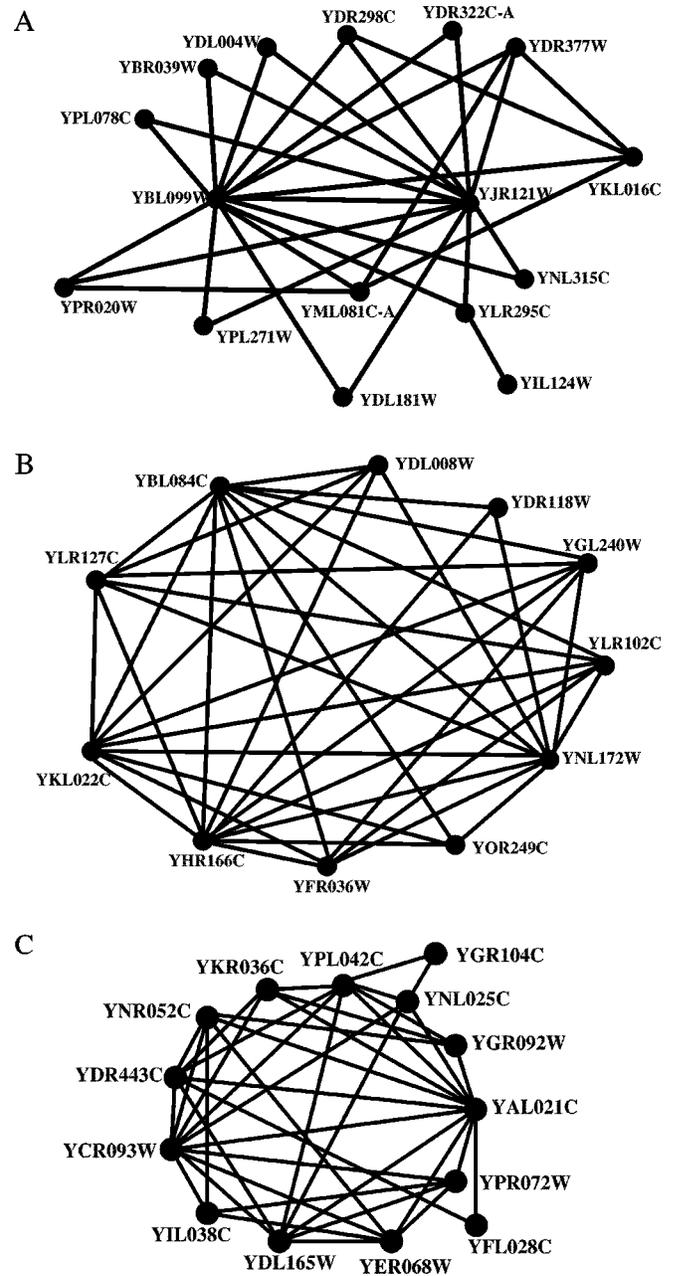


FIG. 7. Examples of proteins clusters identified according to the community structure of Fig. 6.

YFR036W, YHR166C, YKL022C are known to be cell division control proteins; YGL240W plays a role in cell cycle and mitosis; YDR118W, YNL172W, YOR249C probably are membrane proteins; and YLR127C, YDL008W, YLR102C are hypothetical proteins whose functions remain to be determined. It is quite likely that all the proteins in this cluster are closely involved in cell division and membrane fission process. We anticipate that the three hypothetical proteins of this cluster will also have similar biological functions.

The third example is a cluster that appears only when the resolution threshold is refined to below 0.88. It contains 14 proteins and 41 protein-protein interactions. This cluster is also stable and has no further structure. The interaction pat-

tern of this cluster is demonstrated in Fig. 7(c). Among these 14 proteins, according to the protein interaction databank [10,12], YCR093W, YPR072W, YDL165W, YER068W, YIL038C are general negative regulator of transcription subunits; YAL021C is a glucose-repressible alcohol dehydrogenase transcriptional effector; YNR052C is a ubiquitous transcription factor; YDR443C, YGR104C are suppressors of RNA polymerases; YNL025C is the RNA polymerase II holoenzyme cyclinlike subunit; YPL042C is the meiotic mRNA stability protein kinase UME5; YGR092W is the cell cycle protein kinase DBF2; and YKR036C and YFL028C are two hypothetical proteins. It is quite likely that this cluster is mainly involved in RNA transcription process and we also anticipate that the two hypothetical proteins of this cluster are strongly related with this biological function.

To conclude this section, we stress that based on the community structure of Fig. 6 many clusters of proteins can be constructed. Here, we have mentioned just three examples. These identified protein clusters could help researchers to assign possible biological functions to hypothetical proteins, and could also suggest possible proteins that may be involved in carrying out a particular biological reaction.

It should also be emphasized that the protein-protein interaction network is based on high-throughput yeast two-hybrid methods [1]. There may be considerable false-positive and false-negative effects (for a detailed discussion, please refer to Ref. [13]). For example, in Fig. 7(a) protein YBL099W has direct links with many other proteins of this cluster, while according to the protein-protein interaction database at Munich Information center for Protein Sequences (MIPS) [14] only the link with YDR322C-A should be included. The reason why YBL099W appears as interacting with others may be that most of the proteins in this community belong to the ATP synthase complex and therefore they interact in an indirect way (and lead to positive signals in the two-hybrid experiments).

## V. CONCLUSION AND DISCUSSION

In our earlier work [6], the distance between two vertices of a graph is defined as the average number of steps a

Brownian particle takes to move from one vertex to the other. Based on this distance measure, in the present work, we define a dissimilarity index to signify to what extent two nearest-neighboring vertices will be different from each other. We observe that vertices belonging to the same group usually have very small dissimilarity indices between them, while vertices of different communities usually have large dissimilarity indices between them. The observation leads naturally to an algorithm of network clustering. We applied this method to several artificial networks and also to different real networks in social and biological systems and satisfactory results are obtained. Different clusters of a network obtained by our method are characterized by a range of resolution threshold.

The examples studied by us in this paper suggest that our algorithm is very promising in identifying the community structure of a complex network system. Why it works? Maybe it is because of the following reasons. First, the vertex-vertex distance measure has taken into account the topological structure of the network as well as the local connections of the network. The distances from one vertex to all the other vertices of the network actually give a perspective of the whole network viewed from this vertex. Second, the dissimilarity index defined by Eq. (1) compares the perspectives viewed from two nearest-neighboring vertices. It is intuitively appealing to assume that the perspectives of the different vertices of the same community are similar to each other, while those of vertices of different communities will be quite different.

In this paper, we have suggested a powerful metric of similarity. There should be no real difficulty in plugging this metric in traditional hierarchical clustering algorithms to greatly improve their performance.

It is anticipated that the present work will find applications in the field of complex networks, as well as in the fields of sociological and biological sciences.

## ACKNOWLEDGMENTS

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